ESA/Contract No. 4000126561/19/I-NB

Consortium Members

ESA Sea Level CCI+

Algorithm Development Plan

Nomenclature:	SLCCI+_ADP_057_AlgoDevPlan
Issue:	1.0
Date:	Jan. 9, 23

Chronology Issues:			
Issue:	Date:	Reason for change:	Author
1.0	09/01/23	Initial Version	P. Prandi (CLS)

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Acceptance of this deliverable document:			
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FORM-NT-GB-7-1

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1. Introduction

This document provides the algorithm development plan for the ESA sea level CCI+ project addressing the question of local sea level rise uncertainty estimations. This builds on activities performed in the frame of the previous phase of the project where we estimated uncertainties on local sea level trends and accelerations (Prandi et al., 2021) based on a local derivation of the error budget approach used at global scale by Ablain et al. (2019).
It describes a way to generalize the previous study with the inclusion of a spatial dependency of error covariance functions and a way to invert the parameter estimation equations on any (chosen) region of interest.

2. Mathematical statement

The error budget approach is based on the extended least squares formulation. Let **y** be the sea level observations and **X** be a set of coordinates indicating when (or where) these observations are made. The regression model can be written as $y = X\beta + \varepsilon$ where ε denotes deviations from the model and β are the model parameters to be estimated.

If ε follows a normal law of mean 0 and covariance Σ, then the variance of the parameter's estimator is given by

$$\hat{\beta} = N(\beta, (X^t X)^{-1} X^t \Sigma X (X^t X)^{-1})$$

From which the uncertainty on model parameters can be deduced (at any given confidence level).

The error budget is used to build Σ by summing individual contributions from the error terms considered (orbit, wet tropospheric correction, …).

In Ablain et al. (2019) and Prandi et al. (2021) only the time dependency of errors is considered, ie **y** and ε are functions of time only. This implies that the spatial scale of the analysis is driven by the time/space scale at which the error budget was derived:

- 10 days, global in Ablain et al. (2019)
- 1 year, 2° in Prandi et al. (2021)

The algorithm development described in this document aims at addressing this limitation by generalizing to two-dimensional error covariance functions that can be used at any chosen regional analysis scale.

3. Implementation

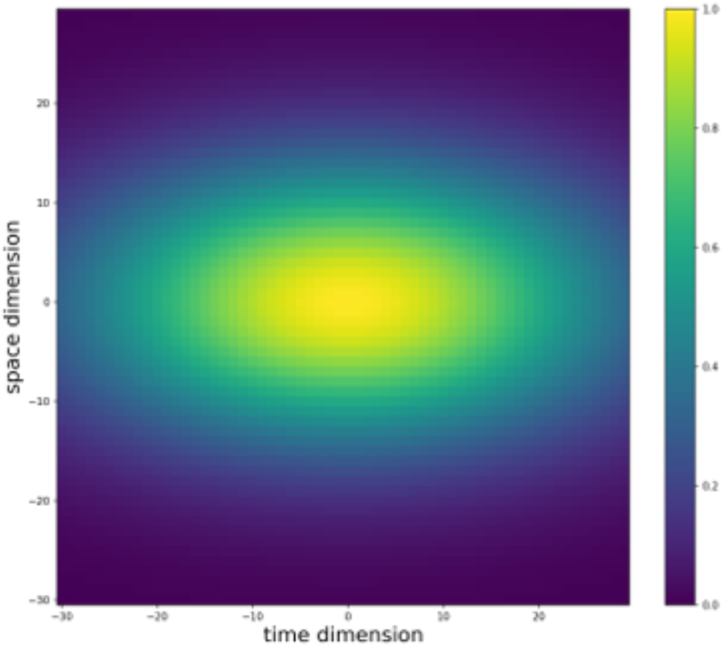
3.1. Two-dimensional covariance functions

The first algorithm change is to implement two-dimensional covariance function estimations, based on existing error covariance models (bias, drift and correlated noise).

We will implement exponentially decaying covariance functions in the space dimension, time-dependent models are left untouched.

This will be implemented under the assumption that time/space covariances are negligible, meaning that $\Sigma_{i,j} = f(t_i, t_j, x_i, x_j)$ can be written as $\Sigma_{i,j} = f_t(i,j) \times f_x(i,j)$.

An example of such covariance functions is given below (for a correlated noise in both time and space):



3.2. Regional inversion

In a second time the inversion method will be updated to consider time and space dimensions in the construction of the least-squares model. This will be done by flattening time and space dimensions prior to building the system design matrix **X.**

Note that the output of this inversion is **one** vector of model parameters (and corresponding variances). The outcome of the model parameter uncertainty estimation is only as good as the error covariance levels and scales that are used as input to the estimation of Σ, and most of the work will be on trying to estimate spatial covariance scales.

4. Schedule

Below is a tentative schedule for the realisation of the algorithm development described in the present plan:

- T0 + 6 months: implementation of two-dimensional error covariance matrices,
- T0 + 9 months: implementation of the regional inversion,
- T0 + 12 months: complete algorithm delivery

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